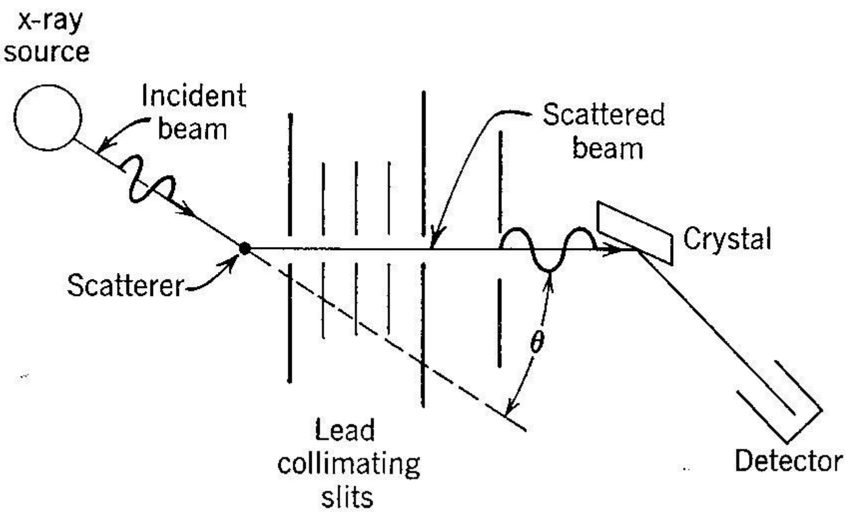
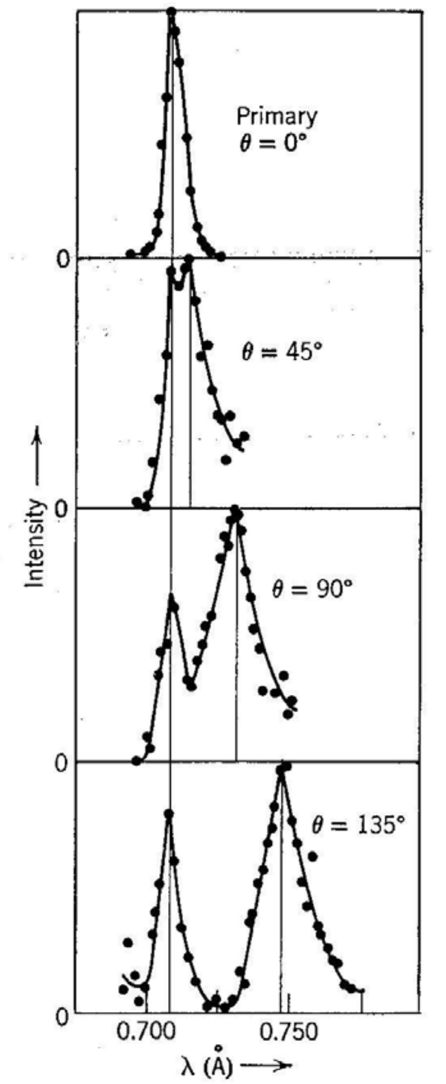
Compton-Effekt





$$\lambda' = \lambda + \lambda_C (1 - \cos\theta)$$

λ_C = Compton-Wellenlänge

$$\text{Expt.:} \quad \lambda_C = \frac{\lambda' - \lambda}{1 - \cos(\theta)} = \frac{(75 - 71)\,\text{pm}}{1.707} = 2.34\,\text{pm}$$

$$\text{Theorie:} \; \lambda_C = \frac{h}{m_e c} = 2.43 \cdot 10^{-12}\,\text{m}$$

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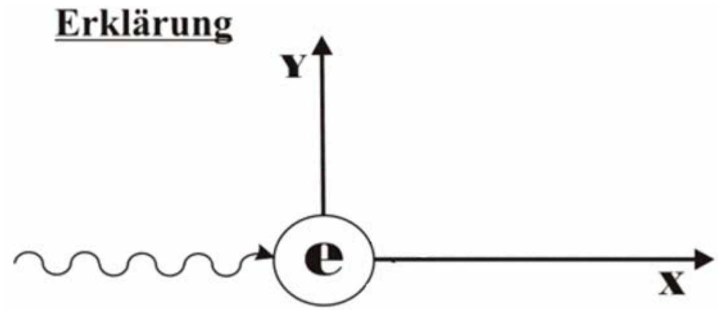

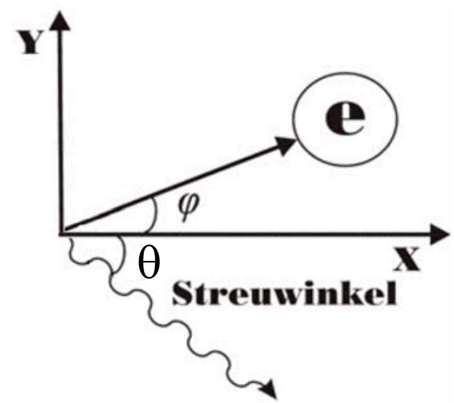

Vor dem Stoß:
e^- in Ruhe $\quad E_{kin} = 0;\ p = 0$

Photon: $\quad E_0 = h\nu;\ p_0 = h\nu / c$

Nach dem Stoß:
e^- in Bewegung $\quad E_{kin} = \frac{1}{2} m_e v^2;\ p = m_e v$

Photon: $\quad E' = h\nu';\ p' = h\nu' / c$

Energieerhaltung: $$h\nu = h\nu' + \frac{1}{2} m_e v^2$$

Impulserhaltung: x-Richtung: $$\frac{h\nu}{c} = \frac{h\nu'}{c} \cos\theta + m_e v \cos\varphi$$

y-Richtung: $$0 = \frac{h\nu'}{c} \sin\theta + m_e v \sin\varphi$$

4 Unbekannte: $\nu',\ \theta, v, \varphi$

Elimination der e^- Größen

$$\Rightarrow \Delta\nu = f(\theta)$$

Compton-Effekt

$$2)\quad \frac{h\nu}{c} - \frac{h\nu'}{c}\cos\theta = m_e v \cos\varphi \Big|^2 \Rightarrow \left(\frac{h\nu}{c}\right)^2 - 2\frac{h^2\nu\nu'}{c^2}\cos\theta + \left(\frac{h\nu'}{c}\right)^2 \cos^2\theta = (m_e v)^2 \cos^2\phi$$

$$3)\quad -\frac{h\nu'}{c}\sin\theta = m_e v \sin\varphi \Big|^2 \Rightarrow \left(\frac{h\nu'}{c}\right)^2 \sin^2\theta = (m_e v)^2 \sin^2\varphi$$

$$(2)^2 + (3)^2 \Rightarrow 4)\ \left(\frac{h\nu}{c}\right)^2 - 2\frac{h^2\nu\nu'}{c^2}\cos\theta + \left(\frac{h\nu'}{c}\right)^2 = (m_e v)^2 \qquad \text{mit } \cos^2\alpha + \sin^2\alpha = 1$$

$$(4)\cdot\frac{1}{2m_e} \Rightarrow 5)\ \frac{h^2}{2m_e c^2}(\nu^2 - 2\nu\nu'\cos\theta + \nu'^2) = \frac{1}{2}m_e v^2$$

$$\text{Einsetzen in }\ 1)\ h\nu = h\nu' + \frac{1}{2}m_e v^2 \Rightarrow h\nu = h\nu' + \frac{h^2}{2m_e c^2}(\nu^2 - 2\nu\nu'\cos\theta + \nu'^2)$$

$$\Delta\nu\ \ \text{klein} \Rightarrow \nu \approx \nu' \text{ in } (...) \Rightarrow h\nu = h\nu' + \frac{h^2\nu^2}{2m_e c^2}\cdot 2\cdot(1-\cos\theta)$$

Compton-Effekt

$$h\nu = h\nu' + \frac{h^2\nu^2}{\cancel{2}m_e c^2}\cdot\cancel{2}\cdot(1-\cos\theta)\,\Big|\div h;\,-\nu'$$

$$\Rightarrow \Delta\nu = \nu - \nu' = \frac{h\nu^2}{m_e c^2}(1-\cos\theta) = \frac{h}{m_e\lambda^2}(1-\cos\theta)$$

$$\text{Mit}\quad \frac{d\nu}{d\lambda} = -\frac{c}{\lambda^2} \;\Rightarrow\; \Delta\lambda = -\frac{\lambda^2}{c}\Delta\nu$$

$$\Rightarrow \Delta\lambda = \lambda - \lambda' = -\frac{\lambda^2}{c}\frac{h}{m_e\lambda^2}(1-\cos\theta) = -\frac{h}{m_e c}(1-\cos\theta)$$

$$\Rightarrow \lambda' = \lambda + \frac{h}{m_e c}(1-\cos\theta) = \lambda + \lambda_C(1-\cos\theta)$$

$$\lambda_C = \frac{h}{m_e c} = \frac{6.63\cdot 10^{-34}\;\cancel{kg}\;m^{\cancel{2}}\;\cancel{s^{-2}}\;\cdot\cancel{s}}{9.1\cdot 10^{-31}\,kg\cdot 3\cdot 10^8\;\cancel{m}\;\cancel{s^{-1}}} = 0.243\cdot 10^{-11}\,m = 2.43\cdot 10^{-12}\,m$$