Übungen zur Physikalischen Chemie für Mikrosystemtechnik WS2007/2008

Übungsblatt 2 – Lösungen

A2.1:.

$$E_{kin} = \frac{1}{2} m v^2 \qquad \langle E_{kin} \rangle = \frac{1}{2} m \langle v^2 \rangle$$

$$\langle v^2 \rangle = \int_0^\infty v^2 f(v)\, dv \quad \Big| \quad \langle E_{kin} \rangle_m = N_A \cdot \frac{1}{2} \cdot m \langle v^2 \rangle = \frac{1}{2} M \langle v^2 \rangle$$

$$f(v) = 4\pi \left(\frac{M}{2\pi RT}\right)^{3/2} v^2 e^{-Mv^2/2RT}$$

$$\langle v^2 \rangle = \int_0^\infty v^2\, 4\pi \left(\frac{M}{2\pi RT}\right)^{3/2} v^2 e^{-Mv^2/2RT}\, dv =$$

$$= 4\pi \left(\frac{M}{2\pi RT}\right)^{3/2} \int_0^\infty v^4 e^{-Mv^2/2RT}\, dv =$$

$$a = \frac{M}{2RT}$$

$$= 4\pi \left(\frac{M}{2\pi RT}\right)^{3/2} \int_0^\infty v^4 e^{-av^2}\, dv =$$

$$= 4\pi \left(\frac{M}{2\pi RT}\right)^{3/2} \cdot \frac{3}{8} \left(\frac{\pi}{a^5}\right)^{1/2} =$$

$$= \frac{3\, M^{3/2} \cdot \pi^{1/2} \cdot 2^{5/2} (RT)^{5/2} \cdot 4\pi}{8\, (2\pi)^{3/2}\, (RT)^{3/2}\, 2^{3/2}\, M^{5/2}} =$$

$$= 3 \cdot \left(2^{-6/2} \cdot 2^{-3/2} \cdot 2^{5/2} \cdot 2^{4/2}\right)\left(M^{3/2} \cdot M^{-5/2}\right)\left(\pi^{1/2} \cdot \pi^{-3/2} \cdot \pi^{2/2}\right)(RT)^{5/2-3/2} =$$

$$= 3 \cdot 1 \cdot M^{-1} \cdot 1 \cdot RT =$$

$$= \frac{3RT}{M} =$$

$$\langle E_{kin} \rangle_m = \frac{1}{2} M \langle v^2 \rangle = \frac{1}{2} M \cdot \frac{3RT}{M} = \frac{3}{2} RT \quad \text{q.e.d.}$$

A2.2: Die Spannung der Ballonhülle wollen wir vernachlässigen.

$p_A V_A = nRT_A, \quad p_E V_E = nRT_E \quad [1.1–1]$

$$\frac{p_A V_A}{nRT_A} = \frac{p_E V_E}{nRT_E} \quad \text{oder} \quad p_E = \left(\frac{V_A}{V_E}\right) \cdot \left(\frac{T_E}{T_A}\right) p_A$$

$$V_E = \tfrac{4}{3}\pi R_E^3, \quad V_A = \tfrac{4}{3}\pi R_A^3, \quad p_E = \left(\frac{R_A}{E_E}\right)^3 \cdot \left(\frac{T_E}{T_A}\right) p_A$$

$R_A = 1$ m, $R_E = 3$ m, $T_A = 298$ K, $T_E = -20\ °C \approx 253$ K, $p_A = 1$ bar.

$$p_E = \left(\frac{1\ \text{m}}{3\ \text{m}}\right)^3 \cdot \left(\frac{253\ \text{K}}{298\ \text{K}}\right) \cdot (1\ \text{bar}) = \left(\frac{1}{3}\right)^3 \cdot (0.849) \cdot (1\ \text{bar}) = \underline{0.03\ \text{bar.}}$$

Mittl. kinetische Energie $\langle E_{kin} \rangle = \frac{3}{2} k_B T = 1.38 \cdot 10^{-23}\ \text{J K}^{-1} \cdot 253\ \text{K} \cdot \frac{3}{2} =$

$= 5.24 \cdot 10^{-21}\ \text{J} \quad (\hat{=}\ 3.15\ \text{kJ mol}^{-1})$

Mittlere Geschwindigkeit:

(1) Billigvariante (nicht ganz korrekt, aber ok, wenn die Studenten so rechnen)

$\langle E_{kin} \rangle = \frac{1}{2} m \langle v^2 \rangle$

$$\langle v \rangle \simeq \langle v^2 \rangle^{1/2} = \left(\frac{2 \langle E_{kin} \rangle}{m}\right)^{1/2} = \left(\frac{2 \cdot 5.24 \cdot 10^{-21}\ \text{kg m}^2 \text{s}^{-2}}{3.32 \cdot 10^{-27}\ \text{kg}}\right)^{1/2} = 1776.4\ \frac{\text{m}}{\text{s}}$$

(≈ 6400 km/h)

$$m_{H_2} = \frac{M_w(H_2)}{N_A} = \frac{2 \cdot 10^{-3}\ \text{kg mol}^{-1}}{6.022 \cdot 10^{23}\ \text{mol}^{-1}} = 3.32 \cdot 10^{-27}\ \text{kg}$$

② korrekte Variante:

$$\langle v \rangle = \int_0^\infty v\, f(v)\, dv = \int_0^\infty 4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} v^3 e^{-\frac{mv^2}{2k_BT}}\, dv$$

Formelsammlung: $\int_0^\infty x^3 e^{-ax^2} = \frac{1}{2a^2}$

$$\langle v \rangle = 4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} \int_0^\infty v^3 e^{-\underbrace{\frac{m}{2kT}}_{\hat{=}\, a} v^2} = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \frac{4k^2T^2}{2m^2} =$$

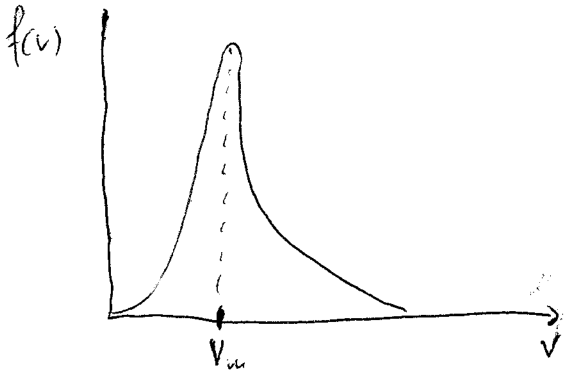
$$= (2\pi)^{-3/2} \cdot 2\pi \cdot 4 \frac{m^{3/2} \cdot m^{-2}}{k^{3/2} k^{-2}\ T^{3/2}\ T^{-2}} = 4 \cdot (2\pi)^{-1/2} \left(\frac{m}{k^{-1/2}T^{-1/2}}\right)^{-1/2} =$$

$$= 4\left(\frac{kT}{2\pi m}\right)^{1/2} = \left(\frac{16\, kT}{2\pi m}\right)^{1/2} = \left(\frac{8kT}{\pi m}\right)^{1/2}$$

$$\langle v \rangle = \left(\frac{8k_BT}{\pi m}\right)^{1/2} = \left(\frac{8 \cdot 1{,}38 \cdot 10^{-23} \cdot 253}{3{,}14 \cdot 3{,}32 \cdot 10^{-27}} \frac{kg\ m^2\ K^{-1}\ K}{s^2\ kg}\right)^{1/2} =$$

$$= 1636{,}4\ \frac{m}{s}$$

A 2.3:



häufigster Geschwindigkeitswert $v_m = 557$ m/s

$\hat{=}$ Maximum von $f(v)$

bei Minimum ä $f(v)$ $\Rightarrow$ $\frac{df(v)}{dv} = 0$ $\rightarrow$ Maximum

$$\frac{d f(v)}{dv} = \frac{d}{dv} 4\pi \left(\frac{M}{2\pi RT}\right)^{3/2} v^2 e^{-av^2} = 0$$

$$= 4\pi \left(\frac{M}{2\pi RT}\right)^{3/2} \left[2v \cdot e^{-av^2} - v^2 \cdot 2av \cdot e^{-av^2}\right] =$$

$$= \text{const.}\ 2v \cdot e^{-av^2} \left[1 - av^2\right] = 0$$

$$\Rightarrow 1 - av^2 = 0 \Rightarrow av^2 = 1 \Rightarrow v_m = \left(\frac{1}{a}\right)^{1/2}$$

$$a = \frac{m}{2k_B T} = \frac{M}{2RT} \Rightarrow v_m = \left(\frac{2RT}{M}\right)^{1/2}$$

$$\Rightarrow v_m^2 = \frac{2RT}{M} \Rightarrow M = \frac{2RT}{v_m^2}$$

$$M = \frac{2 \cdot 2{,}48 \cdot 10^{+3}}{(557)^2} \frac{kg \cdot m^2 \cdot s^2}{s^2 \cdot m^2 \cdot mol} =$$

$$= \frac{10^6 \cdot 2 \cdot 2{,}48}{(557)^2} \frac{g}{mol} = 1{,}599 \cdot 10^{-5} \cdot 10^6 \frac{g}{mol} = 16\ g/mol$$

Bei dem Gas könnte es sich um Methan CH_4 handeln.